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METHODS AND VELOCITY REQUIREMENTS FOR THE RENDEZVOUS OF SATELLITES IN CIRCUMPLANETARY ORBITS

By William E. Brunk and Richard J. Flaherty

Lewis Research Center
Cleveland, Ohio

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E-322

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SUMMARY

In the future of space flight there will be occasions when it is necessary to perform a rendezvous between two satellites in orbit around a planet. For a spherical planet methods of rendezvous are considered in which the total velocity required is kept near a minimum. It is assumed that only one of the satellites is maneuverable and that all velocity increments are applied tangential to the orbit. The problem is further simplified in that the rendezvous takes place outside the atmosphere and the motion of each satellite is determined from a solution of the two-body problem. Two basically different sets of initial conditions are studied. In the first, the maneuverable satellite is launched directly into the rendezvous, while, in the second, both satellites are in orbit before a rendezvous is attempted. Since the Earth, like some of the other planets, is not spherical, the effects of the Earth's oblateness on the methods of rendezvous are presented.

CC-1

INTRODUCTION

One of the problems that will arise in space flight is the deliberate meeting of two or more satellites. Such a meeting is desirable for many reasons. It will be necessary for the transfer of personnel and supplies from the Earth to a semipermanent space laboratory or for the assembling of subsystems for the construction of either a permanent satellite base or large interplanetary expeditions. When manned interplanetary flight becomes a reality, situations may arise where it would be desirable to explore the surface of other planets from smaller space vehicles launched from the large interplanetary vehicle which remains in orbit outside the atmosphere of the planet. Recovery of the exploring vehicles will necessitate a meeting with the orbiting vehicle. In reference 1 it is shown that considerably less total weight is required for a manned interplanetary trip if the crews are returned to the surface of the Earth in small ferrying space vehicles launched from the Earth rather than by landing the interplanetary vehicle.

The type of meeting required for the previous examples will be called rendezvous. It differs from a collision in that ideally there will be no impact. In an ideal rendezvous the orbits of the two vehicles are identical, while in a collision the orbits may be different but must intersect.

Although a rendezvous can be performed between any number of vehicles, the problems are presented more clearly by considering a rendezvous between two vehicles. In this report the problem of rendezvous between two vehicles is presented. Certain assumptions are made which, while limiting the usefulness of the results, enable the basic ideas and methods to be more easily understood. This report is intended to present the general problem rather than a detailed analysis of specific cases. A study of this type gives an indication of both the velocities and times required to perform a rendezvous. This information is useful in estimating propulsion and possibly guidance requirements.

E-322

The problem considered is that of an ideal rendezvous in which the motion of each of the two vehicles is the same as if the other were not present, that is, there is no attraction between the two vehicles. In the actual rendezvous a gravitational attraction exists between the two vehicles which tends to pull them together. This effect is only appreciable when the vehicles are very close to each other and even then is very small. It is also assumed that the vehicles are superimposed when the rendezvous is completed. The rendezvous is to be performed outside the atmosphere, and all velocity increments given are assumed to be exact both in magnitude and direction.

Since the motion of a satellite is determined by the gravity field in which it travels, it is necessary to assume some force field. For simplicity, an inverse-square, central force field is assumed corresponding to a spherical planet. Since this assumption is not exact in the case of the Earth, which has nearly the shape of an oblate spheroid, the effects due to the Earth's oblateness on the presented methods of rendezvous are considered.

There are a number of reports, such as references 2 to 7, which touch on one or more of the ideas presented herein.

RENDEZVOUS

The problems involved in performing an ideal rendezvous are simply stated. It is necessary to perform such maneuvers as required to place two space vehicles at the same location in the same orbit. As no restrictions are placed on the initial positions of the two vehicles, there are an unlimited number of different starting situations. In addition, there are an infinite number of possible transfer trajectories which might be used for any given situation. In order to reduce the total number of possibilities, certain restrictions will be applied herein. These restrictions are:

(1) Only one of the vehicles will be maneuverable.

(2) Only those transfer orbits will be considered which keep the required velocity close to a minimum.

(3) With one exception, only impulsive thrust will be considered.

(4) Except for plane changes, all velocity increments will be applied tangentially.

Restricting the maneuverability to only one vehicle is quite reasonable. If both vehicles were maneuverable, extra propellant would have to be carried by both vehicles and also the orbits of both vehicles would be changed. In the case of smaller transfer vehicles bringing supplies to a space laboratory, the transfer vehicle would do all the maneuvering so that the laboratory would remain in its original orbit. This restriction would be especially important if more than a single rendezvous was to be made.

In any series of transfer orbits taken by the maneuverable vehicle to complete a rendezvous there are basically two variables, velocity increments needed and total time required. In general, these variables are not independent; however, it may be possible to find several values of one for a single value of the other. For example, there may be several transfer trajectories requiring different total times to achieve a rendezvous but all requiring the same total velocity increment. It is possible to choose transfer orbits such that either the total velocity increment or total time is at a minimum. Since, at present, the allowable amount of propellant is critical, transfer orbits for minimum total velocity required are important. For the most part, this is the criterion used herein. However, for certain situations requiring a rendezvous, the transferring of personnel, for example, the total elapsed time is critical. In such cases, transfer trajectories requiring minimum total time might take precedence over minimum-energy trajectories. There are also situations where, because of an upper limit on both the allowable velocity increments and total time, a compromise transfer trajectory must be used.

The transfer trajectory also depends on whether continuous or impulsive thrust is used. Restriction (3) indicates that this study is

E-322

CC-1 back

limited, with one exception, to impulsive thrust. Until continuous-low-thrust devices are developed that are usable in space vehicles such as transfer vehicles, it appears that short-duration rockets will have to be used. A comparison is given herein between continuous thrust and impulsive thrust for rendezvous in one specific case. The comparison shows that in this case the velocity requirement is the highest for continuous thrust.

In general, the impulsive velocity increment could be applied in any direction with respect to the velocity of the space vehicle. However, when the new orbit is to be in the same plane as the original orbit, the velocity increment must be given in that plane. Since velocities add vectorially, the maximum change in resultant velocity for a given velocity increment is obtained when the velocity increment is added tangentially. It can easily be shown that, if it is desired to transfer to an auxiliary orbit to change period or to transfer between two orbits, at least one of which is circular, the smallest velocity increment required is one applied tangentially. Although this result is not always true for a transfer between elliptic orbits, it can be said that, in general, tangentially applied velocity increments are close to the smallest required and for this reason are the only ones considered herein.

In spite of these restrictions, the number of possible initial conditions makes the number of methods of rendezvous very large. In order to reduce the number of cases studied, the initial conditions are divided into two classes. The first of these, to be called direct rendezvous, occurs when the nonmaneuverable satellite is in orbit but the maneuverable satellite has not been launched. The second class, to be called orbital rendezvous, includes all cases where both satellites are in orbit previous to any rendezvous attempt. Since this second class is very large, the method of analysis will be the following. The entire procedure of rendezvous will be presented for certain special initial conditions, while a discussion of possible rendezvous techniques is presented for any initial conditions.

Direct Rendezvous

Basically, direct rendezvous is the simplest of all methods of rendezvous. It is performed by launching the maneuverable satellite into the same orbit as the nonmaneuverable satellite at such a time that the two satellites will be at the same point when the maneuverable satellite enters orbit. This method requires the minimum amount of total velocity, that is, only that velocity required to put the second satellite in the same orbit, and is completed in the shortest possible time. Despite the apparent advantages in this type of rendezvous, difficulty in satisfying the necessary requirements makes it often more difficult to perform a direct rather than an orbital rendezvous.

E-322

Four requirements which must be met in order to perform a direct rendezvous are:

(1) The latitude of the launching site must be no greater than the inclination of the orbit of the nonmaneuverable satellite.

(2) The maneuverable satellite must be launched into an orbit coplanar with that of the nonmaneuverable satellite.

(3) The orbits of both satellites must be identical.

(4) The maneuverable satellite must coincide with the nonmaneuverable satellite at the point where the former enters the orbit.

At any given latitude β, a satellite can be launched into any orbit having an inclination i greater than or equal to β. (All symbols are defined in appendix A.) Thus, orbits having any desired inclination may be initiated from the equator, while only polar orbits are obtainable from polar launching sites. As shown in appendix B, the cost of launching into a circular orbit from a nonequatorial site is no greater than from an equatorial site, provided the inclination of the desired orbit is not less than the latitude of the launching site.

If the orbit of the nonmaneuverable satellite is outside the planet's atmosphere and the planet is a sphere, then the orbit will remain "fixed" in space and the planet will rotate beneath it. However, for a person standing on the planet, the plane of the orbit appears to rotate about the axis of the planet and passes through each point having a latitude no greater than the inclination of the orbit. With one exception, the orbital plane passes through each point at least twice during a sidereal day. A sidereal day is defined as the length of time for the planet to complete a single rotation with respect to the stars. On the Earth, one sidereal day is equal to $23^h56^m4.1^s$ of mean solar time. The one exception to the double passage of the orbital plane is at the point where the latitude is equal to the inclination, and there only a single passage occurs. The time in sidereal days between successive passages is given by

$$\left.\begin{aligned} \Delta t_1 &= \eta/\pi \\ \Delta t_2 &= 1 - \Delta t_1 \end{aligned}\right\} \tag{1}$$

where $\cos \eta = \tan \beta \cot i$. If Δt_1 is the time between the first and second passages, Δt_2 is the time between the second and third passages, and so forth. For an equatorial orbit, $\beta = 0$ so that $\cos \eta = 0$; therefore, $\Delta t_1 = \Delta t_2 = 1/2$. When $i = \beta$, $\cos \eta = 1$ so that $\Delta t_1 = 0$, and there is only a single passage of the orbital plane. A satellite could be launched into this plane every time the plane passes over the launching site. Thus, coplanar launchings can, in general, be carried out twice a day.

To have the orbits of both the maneuverable and nonmaneuverable satellites identical, it is necessary that the maneuverable satellite have the correct velocity and direction at entrance into orbit. If the nonmaneuverable satellite was launched from the same launching site and in the same direction, duplication of the initial entrance conditions would be sufficient. However, if the nonmaneuverable satellite was launched from a different launching site or in a different direction, the desired initial conditions would, in general, have to be calculated.

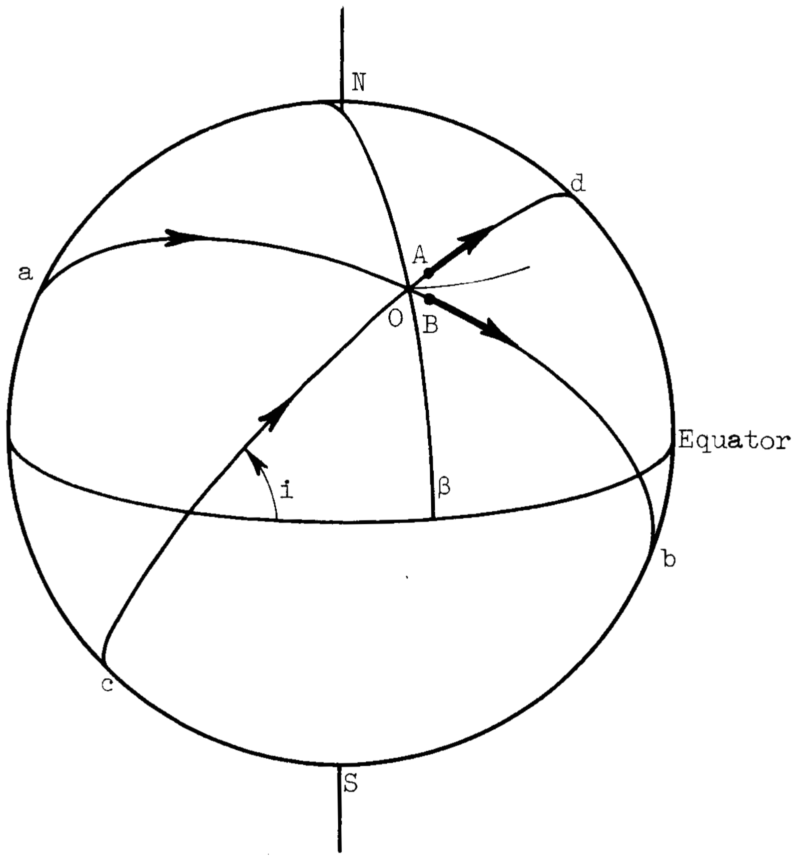
The fourth requirement can be met most simply by predetermining the period P for the nonmaneuverable satellite. Choosing the period determines the size of the semimajor axis since

$$P = \frac{2\pi a^{3/2}}{\mu^{1/2}} \tag{2}$$

where a is the semimajor axis and μ is a constant for each planet. To simplify the analysis, it will be assumed here that all the satellites, including the nonmaneuverable one, are launched from the same site and follow the same launching trajectory. However, the fourth requirement can be met even if these assumptions are not made. The analysis is similar but more involved than that given here.

From the discussion of the second requirement, whenever $i > \beta$ there are two times a day when the maneuverable satellite can be launched into the orbit of the nonmaneuverable satellite. At one time the maneuverable satellite must be launched in a northeasterly direction, and at the other time it must be launched in a southeasterly direction. This is shown in sketch (a) for circular orbits. The sphere is the surface which contains the satellite orbits. The equator and north and south points shown are extensions of the planet's equator and polar axis. The orbit is shown at the two times that it crosses over the launching site at O. The two points A and B are the points where a satellite launched from O with the assumed launching trajectory will enter the orbit. These are called the north and south entrance points, respectively, and for a given orbit are fixed relative to the launching site. The

E-322



(a)

nonmaneuverable satellite is said to pass through an entrance point when it is at point A when on path cd or at point B when on path ab.

In order to perform a direct rendezvous it is necessary for the nonmaneuverable satellite to pass through an entrance point. If the nonmaneuverable satellite passes through one of the entrance points at launching, it will pass again through the same entrance point if it is an orbit chosen to have a period P in sidereal days where

$$P = m/n \qquad (m,n \text{ are integers}) \tag{3}$$

The period is the ratio of the number m of sidereal days to complete n revolutions. All the material given thus far for this requirement is applicable to elliptical as well as circular orbits.

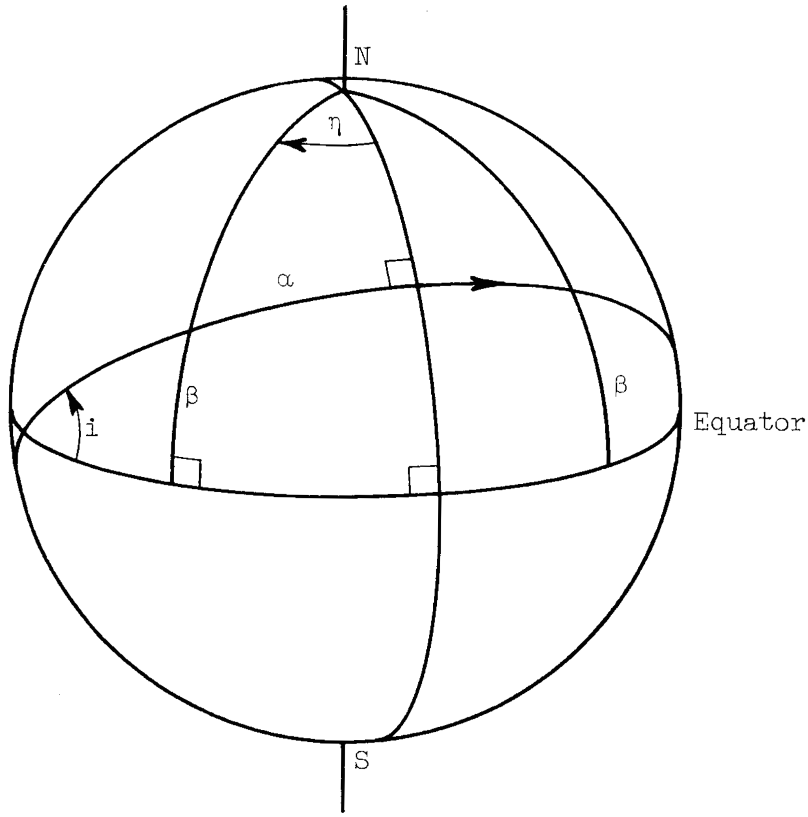
If only circular orbits are being considered, many other periods exist for which a direct rendezvous is possible. If the nonmaneuverable satellite is launched in the northeast direction, entrance point A, in the northern hemipshere, it will pass through entrance point B at some later time if

$$\left(n + \frac{\alpha}{\pi}\right)P = mT + \Delta t_1 \tag{4}$$

where

$$\alpha = \cos^{-1}\left(\frac{\sin \beta}{\sin i}\right) \tag{5}$$

T is one sidereal day and P is in sidereal days. That is, the time required to complete an integral number of revolutions plus that fraction of a revolution to go from latitude β to latitude β again (see sketch (b)) must be equal to an integral number of sidereal days plus the



(b)

time for the Earth to rotate through the angle 2η which from equation (1) is Δt_1. If the nonmaneuverable satellite is launched in the southeast direction, entrance point B, it will pass through entrance point A at some later time if

$$\left(n - \frac{\alpha}{\pi}\right)P = mT - \Delta t_1 \tag{6}$$

If the launching site is in the southern hemisphere, equation (4) applies to a southeast launching and equation (6) to a northeast launching.

Orbital Rendezvous

When both satellites are in orbit before an attempt is made to rendezvous, the technique used will be called orbital rendezvous. If the initial orbit of the maneuverable satellite is in the same plane and entirely within the orbit of the nonmaneuverable satellite, the total velocity cost can, in certain cases, be no greater than for a direct rendezvous. In other cases, especially when the two satellite orbits are not coplanar, the total cost in velocity can be much greater than for a direct rendezvous.

In general, to rendezvous, there are four maneuvers to be considered:

(1) The orbits of the two satellites must be made coplanar.

(2) The orbits must be made tangent at a point.

(3) The periods must be altered so that the satellites meet.

(4) The velocities must be made equal when the satellites meet.

These maneuvers will be discussed separately in the following paragraphs. It should be remembered that in many cases the maneuvers will not be performed separately or even in the order given.

Making Orbits Coplanar

To change the orbit plane of a satellite by an angle γ without altering the orbital speed, it is necessary to apply a velocity increment equal to $2 \sin \frac{\gamma}{2}$ times its velocity perpendicular to the radius at an angle of $90 + \frac{\gamma}{2}^{\circ}$ with respect to this velocity. The velocity requirements for various values of γ are shown in figure 1. These must be applied in a plane normal to the radius vector. There are two points in the orbit where a satellite could, with a single velocity impulse, change from its plane to a desired plane. These are the points where the original orbit intersects the desired plane. If the satellite velocity perpendicular to the radius is lower at one of these points, it is the most economical one at which to make the change unless this makes the following maneuvers more costly. In some cases, it is advantageous to change the orbit in the initial plane before making a plane change.

In this report, velocity increments in maneuvers except plane changes are added tangentially, and only tangent transfer orbits are considered. Since velocities add vectorially, the biggest change in magnitude for a given velocity difference Δv occurs when Δv is added or subtracted in the same direction as the original velocity. A rendezvous can be performed in less time by using nontangential transfer orbits, but these generally become costly in power because of the direction changes involved. This cost in velocity is $2 \sin \frac{\gamma}{2}$ times initial speed if the initial speed is kept constant. A $5^{\circ}44'$ direction change requires an additional velocity 0.1 times the initial velocity.

Making Orbits Tangent

When the orbits are coplanar, it is necessary to change the orbit of the maneuverable satellite to one tangent to that of the nonmaneuverable satellite. Some general equations that are very useful are:

(1) Semimajor axis of an orbit:

$$a = \frac{\mu}{2 \frac{\mu}{r} - v^2} \tag{7}$$

(2) Velocity for a circular orbit:

$$v_c = \sqrt{\frac{\mu}{r}} \tag{8}$$

(3) Equation of orbit in polar coordinates:

$$\frac{1}{r} = \frac{\mu}{h^2}\left[1 + \sqrt{1 + \frac{2\left(\frac{v^2}{2} - \frac{\mu}{r}\right)h^2}{\mu^2}} \cos \varphi\right] \tag{9}$$

Since h, the angular momentum, and $\frac{v^2}{2} - \frac{\mu}{r}$, the total energy, are constant for an orbit, all that are necessary to determine an orbit are the velocity vector and radius vector at one point. The angular momentum h is equal to the product of the radial distance times the velocity perpendicular to the radius.

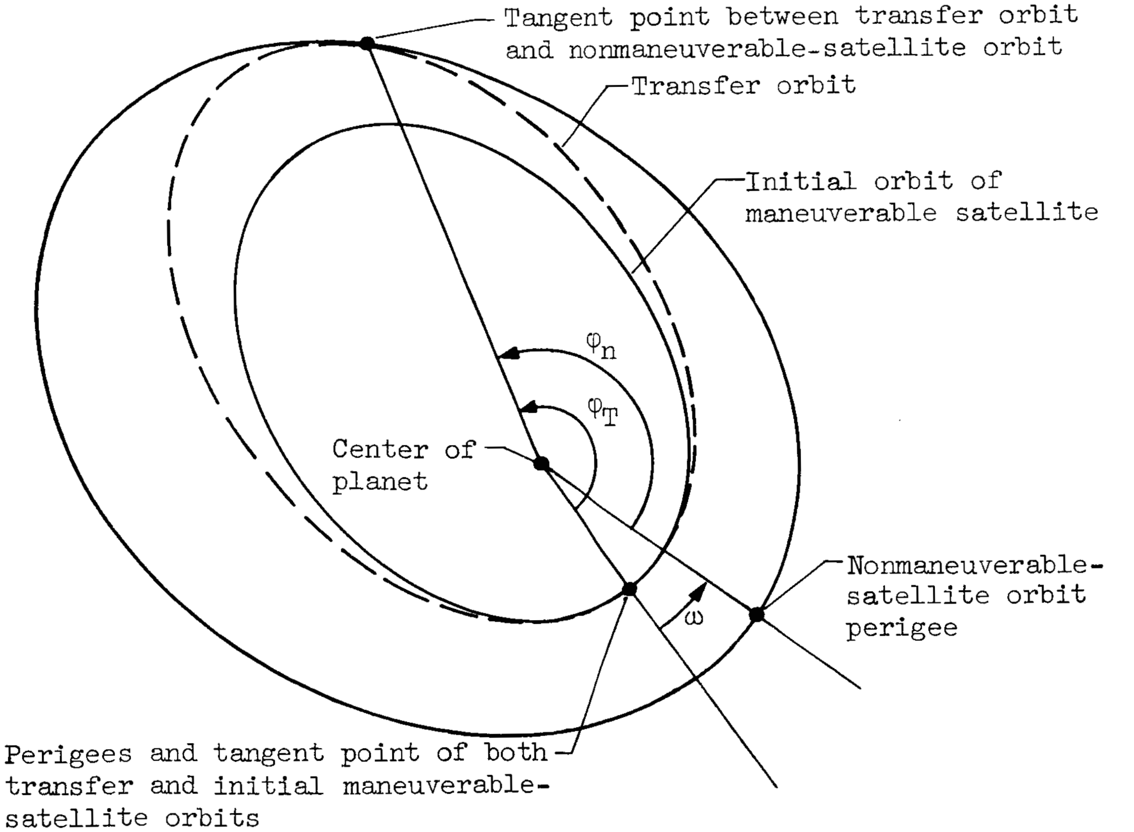
If the satellites are in two arbitrary elliptical orbits, it is difficult to find an explicit solution for the velocity increment to be applied tangentially at some point on the maneuverable-satellite orbit that would put the satellite into a transfer orbit tangent to the desired orbit. If the maneuverable-satellite orbit were entirely inside or entirely outside the orbit of the nonmaneuverable satellite, the tangential transfer orbit could be started at any point in the maneuverable-satellite orbit. However, if the orbits are intersecting, the transfer can be made only from certain portions of the maneuverable-satellite orbit. The total velocity requirement for the rendezvous is dependent on the choice of the point from which to start the transfer orbit.

For a given point on the maneuverable-satellite orbit, a tangential transfer orbit can be found by varying the magnitude of the tangentially applied velocity increment. Properties of orbits that are useful are:

(1) Any two Keplerian orbits, conic sections, cannot intersect at more than two points unless the orbits are identical.

(2) If two orbits have only one intersection point and at least one of the orbits is elliptical, the orbits are tangent at this point.

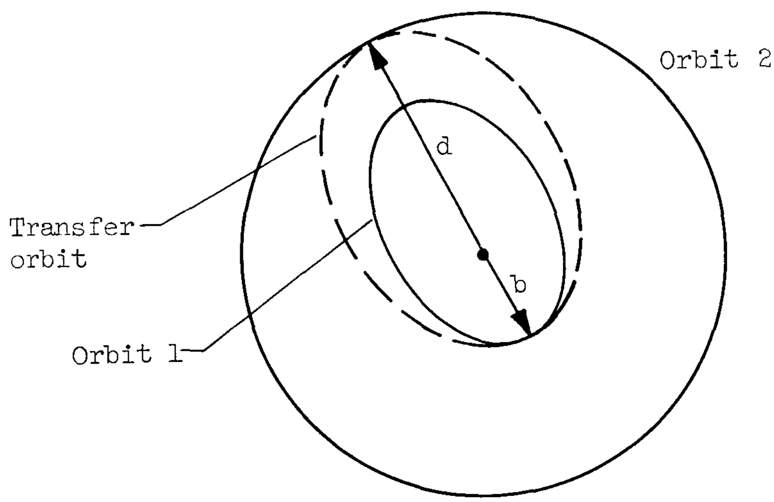
The desired velocity increment gives a real single-valued solution for φ_T, the angular distance from the perigee of the transfer orbit, when equating the right side of the orbit equation (9) for the transfer orbit to the right side of the same equation for the nonmaneuverable-satellite orbit. To account for the orientation of these orbits, $\varphi_T - \omega$ must be substituted for φ_n, the angular distance from the perigee of the nonmaneuverable satellite. Here ω is the angular distance from the perigee of the transfer orbit to the perigee of the nonmaneuverable-satellite orbit (see sketch (c)).



(c)

If the tangential velocity is varied at one of the apsides (perigee or apogee), ω will stay constant unless the velocity is varied through circular velocity. In the latter case, there would be a 180° change in ω, that is, what was the perigee point would become the apogee or vice versa. If the velocity is not varied at an apsis, ω must be recalculated for each velocity increment tried. When the tangential transfer orbit is to be initiated at an apsis (see sketch (c)), a good velocity to start the solution with would be the velocity which would put the other apsis on the nonmaneuverable-satellite orbit. This velocity can be easily obtained (see next paragraph) and would give a tangential transfer orbit if $\omega = 0^{\circ}$ or 180°. If $\omega \neq 0^{\circ}$ or 180°, this velocity gives a velocity slightly higher than that required for a tangential transfer orbit. The correct velocity for the transfer can be found by reducing this velocity until a real single-valued solution for φ_T is found. For special cases, explicit solutions for the velocity requirements to transfer between orbits can be determined.

If the major axes of the maneuverable- and the nonmaneuverable-satellite orbits fall on the same line ($\omega = 0^{\circ}$ or 180°), the velocity requirement can be determined from equation (7). If at least one of the orbits is circular, this requirement is met automatically. Let a_T be the major axis of the tangent transfer orbit (see sketch (d)).



(d)

From equation (7)

$$a_T = \frac{b + d}{2} = \frac{\mu}{\frac{2\mu}{b} - v_{T,b}^2}$$

E-322

E-322

Solving for $v_{T,b}$ gives

$$v_{T,b} = \sqrt{\mu\left(\frac{2}{b} - \frac{1}{a_T}\right)} \tag{10}$$

The Δv to be added at distance b to change the satellite from orbit 1 to the transfer orbit is

$$\Delta v_b = v_{T,b} - v_{1,b} \tag{11}$$

Similarly, the Δv which would put the satellite in orbit 2 into the transfer orbit at distance d is

$$\Delta v_d = v_{T,d} - v_{2,d}$$

where

$$v_{T,d} = \sqrt{\mu\left(\frac{2}{d} - \frac{1}{a_T}\right)}$$

The equations are general, and in sketch (d) orbits 1 and 2 could also be drawn as intercepting, with the circular one on the inside, or with both elliptic with their major axes on the same line. However, b and d must always be in opposite directions on the major axis of the transfer orbit.

In transferring between elliptical and circular orbits, the sum of the two velocity increments will be a minimum if the following rules are adhered to:

(1) If the apogee of the elliptical orbit is less than the radius of the circular orbit, the transfer should be made to or from perigee.

(2) If the apogee is greater than the radius of the circle, the transfer should be made to or from apogee.

Altering the Period to Make Satellites Meet

When the orbits of the maneuverable and nonmaneuverable satellites are tangent, it is necessary to bring the two satellites together at the point of tangency. For example, suppose that at some time the maneuverable satellite will pass through the tangent point 3 minutes after the nonmaneuverable satellite. If, at the time it passes the tangent point, the maneuverable satellite is put into a new orbit with a period 3 minutes less than that of the nonmaneuverable satellite, both satellites will meet on their next pass through the tangent point. If the new period were 1.5 minutes less than that of the nonmaneuverable satellite, the two satellites would meet after two revolutions. This can be expressed as

$$P_n - P_T = \frac{\Delta t}{N_T} \tag{12}$$

where P_T is the period of the new transfer orbit to make up the time difference Δt in N_T revolutions. The time difference Δt is taken as positive if the nonmaneuverable satellite leads the maneuverable satellite past the tangent point.

If two satellites are in tangential but not identical orbits, their periods will be different. In this case, the total velocity required to complete the rendezvous can be made as low as that required to transfer between the orbits. The satellites should remain in their initial tangential orbits until the longer period satellite is in the lead with the difference in time of passage of the tangent point less than the difference in periods.

Since the period of the maneuverable satellite must be altered at the time it passes the tangent point, it is necessary to know in advance the value of Δt. In order to obtain Δt, the location of each satellite in its orbit must be known at some specific time. The time for each of the satellites to travel from this location to the tangent point can be obtained from a knowledge of the time required to reach each of the points from the perigee point. The time of flight from perigee to a point at distance r is

$$t = \frac{r_P^{3/2}}{\sqrt{\mu}\left(2 - \frac{v_P^2 r_P}{\mu}\right)^{3/2}} \left[\sin^{-1} \frac{\left(2 - \frac{v_P^2 r_P}{\mu}\right)\frac{r}{r_P} - 1}{\frac{v_P^2 r_P}{\mu} - 1} + \frac{\pi}{2}\right] \mp$$

$$\frac{r_P^{3/2}\sqrt{\left(\frac{r}{r_P}\right)^2 \left(\frac{v_P^2 r_P}{\mu} - 2\right) + 2\frac{r}{r_P} - \frac{v_P^2 r_P}{\mu}}}{\sqrt{\mu}\left(2 - \frac{v_P^2 r_P}{\mu}\right)} \tag{13}$$

where the - is used up to the apogee point and + is used after apogee.

In terms of the periods of the maneuverable and transfer orbits, the Δv to be added at the point of tangency to alter the period is

E-322

from equations (2) and (7) where r is the distance of the tangent point:

$$\Delta v = \sqrt{\frac{2\mu}{r} - \left(\frac{2\pi\mu}{P_T}\right)^{2/3}} - \sqrt{\frac{2\mu}{r} - \left(\frac{2\pi\mu}{P_m}\right)^{2/3}} \tag{14}$$

Rendezvous Between Satellites in Circular Orbits

In the special case of going from one circular orbit to another, it is possible to wait for the correct angular relation before entering the transfer ellipse so that the satellites will meet on the first pass of the maneuverable satellite through the tangent point. For a transfer from the inner to the outer orbit, the outer satellite must lead the inner satellite by the angle

$$\Theta_{1,2} = 180\left[1 - \left(\frac{1+\rho}{2\rho}\right)^{3/2}\right], \text{ deg} \tag{15}$$

when the maneuverable satellite enters the transfer orbit. Here, ρ is the ratio of the outer to the inner orbital radii. In the reverse case, transfer from the outer to the inner orbit, the outer satellite must lead by an angle

$$\Theta_{2,1} = 180\left[\left(\frac{1+\rho}{2}\right)^{3/2} - 1\right], \text{ deg} \tag{16}$$

when the maneuverable satellite enters the transfer orbit. In terms of the inner circular orbit velocity $v_{c,1}$, the magnitudes of the velocity increments to change between the circular orbits and the transfer orbit are, for the inner orbit,

$$\frac{\Delta v_1}{v_{c,1}} = \Delta V_1 = \sqrt{\frac{2\rho}{1+\rho}} - 1 \tag{17}$$

and, for the outer orbit,

$$\frac{\Delta v_2}{v_{c,1}} = \Delta V_2 = \sqrt{\frac{1}{\rho}}\left(1 - \frac{2}{1+\rho}\right) \tag{18}$$

Equations (15) to (18) are derived in appendix C.

The quantities ΔV_1, ΔV_2, $\Theta_{1,2}$, and $\Theta_{2,1}$ are plotted in figure 2 as a function of the radius ratio ρ. At radius r_1, ΔV_1 is added to go from the circular to the transfer orbit or subtracted to go from the transfer to the circular orbit. Likewise, at radius r_2, ΔV_2 is subtracted to go from the circular to the transfer orbit or added to go from the transfer to the circular orbit.

From figure 2 it can be seen that it is more costly in velocity to transfer a satellite between circular orbits with radius ratios greater than about 3.2 than it is to escape the gravitational field of the planet from the inner orbit since the escape velocity is only $\sqrt{2}$ times the circular velocity.

Rendezvous Between Satellites in the Same Circular Orbit

If two satellites happen to be in the same circular orbit, two methods can be used to rendezvous them. One method is to alter the period of the maneuverable satellite as was described for tangent orbits. For two satellites in the same circular orbit, $\Delta t = P_n \frac{\Theta}{360}$. The angular separation between the satellites Θ is considered positive when measured from the maneuverable to the nonmaneuverable satellite in the direction of revolution. Substituting for Δt in equation (12) gives

$$\frac{\Theta}{360} = N_T\left(1 - \frac{P_T}{P_n}\right) \tag{19}$$

Substituting equations (7) and (2) in the form

$$P = \frac{2\pi\mu}{\left(2\frac{\mu}{r} - v^2\right)^{3/2}}$$

gives

$$\frac{\Theta}{360N} = 1 - \left[\frac{2v_c^2 - v_c^2}{2v_c^2 - (v_c + \Delta v)^2}\right]^{3/2}$$

This can be written as

$$\frac{\Theta}{360N} = 1 - \left[\frac{1}{1 - 2\,\Delta V - \Delta V^2}\right]^{3/2} \tag{20}$$

where

$$\Delta V = \frac{\Delta v}{v_c}$$

or

$$\Delta V = \sqrt{2 - \left(\frac{1}{1 - \frac{\Theta}{360N}}\right)^{2/3}} - 1 \qquad (21)$$

This equation is plotted in figure 3. The magnitude of the negative velocity increments is limited to that which would lower the perigee point to the atmosphere.

Since the velocities of the satellites need to be the same when they meet, another Δv, equal but opposite, is required, which makes the total cost $2\ \Delta v$. The total required velocity can be very small by making the number of revolutions N large. A dimensional plot of this is shown in figure 4 for a 300-mile orbit about the Earth.

The second method is to change the circular velocity with an impulse and then balance the excess or defect in centrifugal force with a continuous radial thrust to keep the maneuverable satellite in the same circular orbit while it is catching up to the nonmaneuverable satellite ($+\Delta v$) or while the nonmaneuverable satellite is catching up to it ($-\Delta v$). This method is often considered but is much more costly in velocity.

The velocity cost of initiating and stopping the maneuver is $|2\ \Delta v|$. The cost of holding the vehicle in the same orbit can be expressed as a velocity because it is an acceleration for a length of time. Centrifugal acceleration CA is proportional to velocity squared if the radius is constant. Since the vehicle is orbiting, $CA = g_r = v^2/r$:

$$CA + \Delta CA = g_r\left(\frac{v_c + \Delta v}{v_c}\right)^2$$

The excess or defect $\Delta CA = g_r\left[\frac{2\ \Delta v}{v_c} + \left(\frac{\Delta v}{v_c}\right)^2\right]$ where $g_r = g_R\left(\frac{R}{r}\right)^2$.

The length of time this acceleration would be needed is equal to the distance divided by the rate of closure:

$$\text{Time} = \frac{D}{\Delta v}$$

Time multiplied by acceleration is equivalent to linear velocity for calculating rocket propellant ratios so that the velocity equivalent of balancing the ΔCA is

$$g_r\left[\frac{2\ \Delta v}{v_c} + \left(\frac{\Delta v}{v_c}\right)^2\right]\frac{D}{\Delta v} = g_r\left(\frac{2}{v_c} + \frac{\Delta v}{v_c^2}\right)D$$

Thus, the maneuver requirement in linear velocity is

$$\text{Requirement} = |2\,\Delta v| + g_r\left(\frac{2}{v_c} + \frac{\Delta v}{v_c^2}\right)D \qquad (22)$$

Letting $g_r = \mu/r^2$ and $r = \mu/v_c^2$ gives

$$g_r = v_c^4/\mu$$

and

$$D = \frac{\pi|\Theta|}{180}\,r = \frac{\pi|\Theta|}{180}\,\frac{\mu}{v_c^2}$$

for Θ in degrees. Substituting into equation (22) and dividing by v_c yield

$$\frac{\text{Velocity requirement}}{v_c} = \left|\frac{2\,\Delta v}{v_c}\right| + \left(2 + \frac{\Delta v}{v_c}\right)\frac{\pi|\Theta|}{180} \qquad (23)$$

This equation is plotted in figure 5. A dimensional plot is given in figure 6 for a 300-mile-altitude orbit about the Earth. Comparing velocity requirements of figure 5 with those of figure 3 shows the greater velocity needed in the method balancing the centrifugal force.

General Remarks on Orbital Rendezvous

It should be remembered that in an actual rendezvous one would combine plane changes with changes in the orbit whenever it would reduce the total rendezvous velocity requirement. Also, it is best to launch the maneuverable satellite into an orbit that is in the same plane and tangent to the perigee point but otherwise entirely within the orbit of the nonmaneuverable satellite. In this case, it is possible to make an orbital rendezvous without exceeding the total velocity required for a direct rendezvous.

EFFECTS DUE TO OBLATENESS

In the previous sections of this report it has been assumed that the planets are spheres and, therefore, that the gravitational attraction acting on the satellites is an inverse-square central force. However, because of rotation about their own axes some of the planets have the form of an oblate spheroid. An oblate spheroid is the solid body formed when an ellipse is rotated about its minor axis. A measure of this "flattening" of the planet is its oblateness, which is defined as the ratio of the difference between the equatorial and polar radii to the equatorial radius. The value of oblateness for the planets ranges from zero for Mercury and Venus to 1/9.5 for Saturn. The oblateness of the Earth is 1/297.0.

E-322

The motion of a satellite in the gravitational field of an oblate body is different from its motion about a sphere. The effects of the Earth's oblateness on the motion of a satellite have been considered in several reports (refs. 8 to 13). Similar studies can be made for orbits about the other planets.

In the remainder of this report, the effect of oblateness on the results obtained in the previous sections will be considered. The discussion will be limited to the case of the Earth. Since many of the papers dealing with motion about the oblate Earth are limited in types and positions of orbits considered, the material used herein was taken from reference 8. In this reference, the general case is solved by a method of perturbations for an orbit with eccentricity less than 0.05 and any inclination. The principal assumption made in reference 8, other than that of limited eccentricity, is that atmospheric effects are neglected. This is compatible with the assumption that the rendezvous will take place outside the atmosphere. The assumption of small eccentricities is the most limiting, but it is indicated in the reference that similar effects due to oblateness are expected in orbits with larger eccentricities.

CC-3 back

The motion of a satellite is determined by the local gravitational field. The gravitational potential at a distance r from the center of a sphere is

$$U = g\frac{R^2}{r} \tag{24}$$

while the potential at a distance r from the center of the Earth can be approximated by

$$U = gR\left[\frac{R}{r} + J\frac{R^3}{r^3}\left(\frac{1}{3} - \cos^2\theta\right) + \frac{F}{35}\frac{R^5}{r^5}\left(35\cos^4\theta - 30\cos^2\theta + 3\right)\right] \tag{25}$$

where

$$J = 1.637\times10^{-3}$$

$$F = 10.6\times10^{-6}$$

$$g = 32.146 \text{ ft/sec}^2$$

$$R = 3963.26 \text{ miles}$$

While equation (24) leads to an inverse-square central force, differentiation of equation (25) gives a component of force in both the radial and tangential directions. This noncentral force field leads to four important variations in the motion of a satellite as compared with the motion in an inverse-square central force field. It is these variations that must be considered in the problem of rendezvous. The effects of these variations on the ideal rendezvous will be given both for general rendezvous and for the specific example of a rendezvous between a satellite initially in a circular orbit at a radial distance of 4200 miles with a satellite in a circular orbit at a distance of 4300 miles. Actually, as will be shown later, circular orbits are not possible for orbits with inclinations greater than zero. The circular orbits referred to in this section of the report are as close to true circular orbits as possible about an oblate planet. The expressions given herein for the variations are correct up to terms of order J^2.

E-322

The first of these variations is a change in the period. The period, if defined as the time elapsed between consecutive northward crossings of the Equator, is

$$P = \frac{2\pi h^3}{g^2R^4}\left(1 - \frac{Jg^2R^6}{h^4}\,\frac{11\cos^2 i - 5}{2} + \frac{3e^2}{2}\right) \tag{26}$$

where h is the angular momentum per unit mass. Equation (26) indicates a decrease in period of amount

$$\Delta P = \frac{2\pi JR^2}{h}\,\frac{11\cos^2 i - 5}{2} \tag{27}$$

compared with the period for motion about a sphere. The change in period ΔP is plotted in figure 7(a) as a function of i and r for a circular orbit. For the specific rendezvous considered, the relative change in periods due to oblateness is shown in figure 8(a). This difference is to be subtracted from the difference in period (199 sec) due to the different radial distances of the two satellites. The maximum change in the relative periods is less than one-fifth of a percent of the total difference in period. This change can therefore be neglected in cases of rendezvous requiring total times equal to a few periods if the satellite orbits are originally within approximately 100 miles of each other. The relative change in period becomes vanishingly small for orbits with inclinations between 45° and 50°.

In order to examine the other differences between satellite motion about an oblate body and motion about a sphere, it is convenient to think of both orbits as lying in a plane. The orbital plane for motion about a sphere is fixed. The second variation in the orbit due to oblateness is a rotation of the orbital plane about the Earth's axis. It is to be remembered that, although the orbital plane rotates, the angle of inclination of the plane remains constant. The mean rate of rotation in degrees per day is

$$\frac{d\Omega}{dt} = \frac{360Jg^2R^6}{Ph^4} \cos i \tag{28}$$

where P is in days. The direction of the rotation is opposite the direction of revolution of the satellite in its orbit. The rate of rotation is plotted in figure 7(b) as a function of i and r for circular orbits.

Although the orbital plane rotates, it is still possible to launch satellites in orbits that are at some time coplanar. The instantaneous plane of the orbiting satellite will still pass over the launching pad at least once a day. The time between passages can be computed allowing for the rotation of the orbital plane. Since the rate of rotation of the orbital plane depends on both the period and the angular momentum, it will be different for satellites at different mean altitudes but with the same angle of inclination. It is the difference in rate of rotation that is important in rendezvous. Figure 8(b) shows this difference for the specific example considered. This relative rotation is greatest for an equatorial orbit where it is about 0.65° per day and reduces to zero for a polar orbit. The relative rotation of 0.65° per day in this example means a separation of about 48 miles per day or 3 miles per period. This can be considered as a maximum value for two orbits differing in altitude by 100 miles as the rate of rotation also decreases with increasing altitude. Thus, only if the radial distances are greatly different will the difference in rate be appreciable. If the rendezvous is to take place between satellites in the same orbit or if the rendezvous is to take place very shortly after the maneuverable satellite is in orbit, the effect of orbital rotation can be assumed negligible. If, on the other hand, the rendezvous is between satellites in orbits with greatly differing radial distances or will take place long after the satellites are put into orbit, the difference in the orbital plane rotation rates can be anticipated, and the maneuverable satellite can be launched in a plane such that the two orbits will be coplanar at the time of rendezvous.

The third variation is a rotation of the direction of the major axis which is not present for motion about a sphere. The rate of rotation in degrees per day is

$$\frac{d\delta}{dt} = \frac{180Jg^2R^6}{Ph^4} (5 \cos^2 i - 1) \tag{29}$$

where P is in days. It should be noticed that since this rotation is proportional to $5 \cos^2 i - 1$ there will be no rotation for $\cos^2 i = 1/5$, $i = 63.4^\circ$; and the rotation will be in the opposite direction for greater i. The direction of rotation for $i < 63.4^\circ$ is in the same direction as the motion of the satellite in its orbit. A plot of $d\delta/dt$ is given in figure 7(c) as a function of i and r for circular orbits.

E-322

The relative rate of rotation for the specific example considered is shown in figure 8(c). The magnitude of the effect is larger than that of the rotation rate of the plane and therefore must be taken into consideration in rendezvous computations. This effect enters the problem of rendezvous principally in cases where the approach satellite is in an elliptical orbit tangent to the circular orbit of the orbiting satellite. Since the major axes are rotating, the point of tangency is moving.

The fourth and final effect due to oblateness is a variation in the radial distance. The radial distance is given by

$$\frac{1}{r} = \frac{gR^2}{h^2} \left[1 + e \cos(\psi - \lambda) + \frac{Jg^2R^6}{h^4} \left(\frac{5 \cos^2 i - 3}{2} + \frac{\sin^2 i}{6} \cos 2\psi\right) + \frac{5Je}{24} \frac{g^2R^6}{h^4} \sin^2 i \cos(3\psi - \lambda)\right] \tag{30}$$

where ψ is the angular distance from the point of maximum latitude in the northern hemisphere and λ is the angle ψ measured to the perigee point. The variation in radial distance for a circular orbit at an assumed radial distance of 4400 miles is given in figure 9 as a function of ψ for various values of i. The only case for which an actual circular orbit is possible is for i equal to zero. However, if two satellites have the same effective latitude, angular momentum, and radial distance at a single given value of ψ, their orbits will be identical. In general, the effect of variation in radial distance can be neglected in rendezvous.

Thus, it is seen that while some of the effects due to the oblateness of the Earth are small, they must be taken into consideration in attempting a rendezvous. Similar effects will be observed when interplanetary vehicles are put into orbit about the other planets. The magnitude of the effects will increase with increasing oblateness. However, for planets with large values of oblateness such as Jupiter, Saturn, Uranus, and Neptune, the effects due to oblateness may be considerably different from those mentioned for the Earth.

E-322

CONCLUSIONS

A general study of the problems involved in performing a rendezvous between two satellites or other space vehicles in orbit around a planet is presented. The problem is simplified by assuming that the motion of each satellite is obtained by a solution of the two-body problem, that only one of the vehicles is maneuverable, and that all velocity increments other than for plane changes are added tangentially. Rendezvous performed outside the atmosphere is the only case considered. The results are for orbits around a spherical planet; however, the effects on these results due to an oblate planet are discussed. The oblateness used is that of the Earth. The following conclusions were reached.

The most efficient method of rendezvous in terms of both total velocity and time required is performed by launching the maneuverable vehicle at such a time and with the necessary initial conditions that the rendezvous is completed the instant the vehicle enters orbit. This type of rendezvous is called direct rendezvous.

Direct rendezvous is limited in usefulness because of the rigid requirements on times of launching of the maneuverable vehicle and the possible periods for the nonmaneuverable vehicle.

Orbital rendezvous, where both vehicles are initially in orbit, is not limited by the rigid requirements of direct rendezvous and is therefore of great importance.

If possible, the initial orbits should be coplanar before an orbital rendezvous is attempted. Plane changes require large velocity increments and should be avoided.

A greater total velocity increment is required to transfer a vehicle between circular orbits with radius ratios greater than about 3.2 than to follow an escape trajectory from the inner orbit.

The total velocity increment required to bring together two vehicles initially in the same orbit can be made very small if the time allowed for the maneuver is not critical.

If the planet has the shape of an oblate spheroid rather than that of a sphere, certain variations occur in the relative orbits of the vehicles. The variations, although small in certain cases, must be taken into consideration in attempting a rendezvous.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, July 16, 1959

APPENDIX A

SYMBOLS

a	semimajor axis
b	defined in sketch (d)
CA	centrifugal acceleration
D	distance
d	defined in sketch (d)
e	eccentricity
F	10.6×10^{-6}
G	universal gravitational constant
g	local constant of gravity
h	angular momentum per unit mass
i	inclination of orbit
J	1.637×10^{-3}
M	mass of planet
m	integer
N	number of revolutions
n	integer
$\dot{P}$	period
R	equatorial radius of planet
r	radial distance
T	sidereal day
t	time
U	gravitational potential

E-322

V	dimensionless velocity
v	velocity
α	defined by eq. (5)
β	latitude of launching site
γ	angle between initial and final orbital planes
δ	angle through which major axis has rotated in orbital plane
ζ	angle between east and launching direction of satellite
η	defined by eq. (1)
Θ	angular separation between satellites as seen from center of planet
θ	angle between rotational axis and radial line
λ	angle between point of maximum latitude in northern hemisphere and perigee point
μ	GM
ξ	angle between east and direction of orbit over launching site
ρ	ratio of outer to inner radii of circular orbits
φ	angular distance of satellite from perigee point
ψ	angular distance of satellite from point of maximum latitude
Ω	angle through which orbital plane has rotated

Subscripts:

b	at distance b
c	circular
d	at distance d
m	maneuverable satellite
n	nonmaneuverable satellite

E-322

CC-4

P	perigee
R	at surface of planet
r	at distance r
T	transfer orbit
1	orbit 1
2	orbit 2

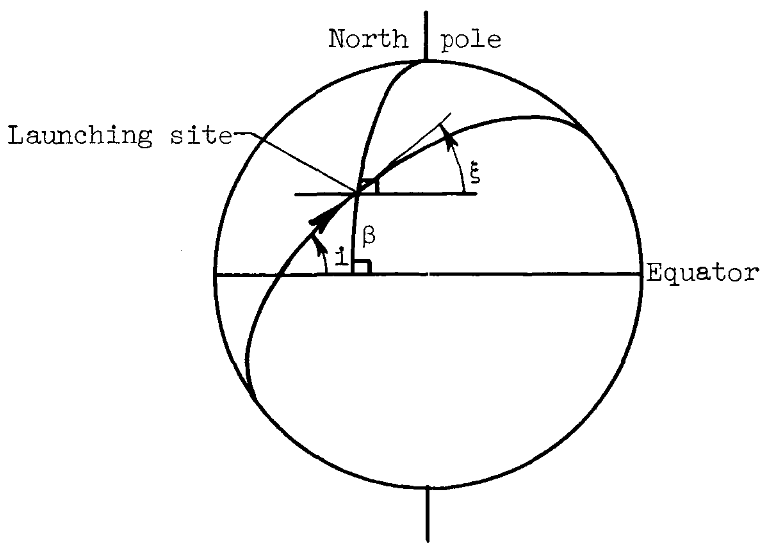
APPENDIX B

BURNOUT VELOCITY REQUIRED TO PLACE A SATELLITE IN A GIVEN CIRCULAR ORBIT WHEN LAUNCHED FROM A SITE OFF THE EQUATOR

It can be shown that no increase in burnout velocity is necessary when the launching site is off the equator for circular orbits with angle of inclination greater than zero. The latitude of the launching site must, however, be no greater than the angle of inclination of the orbit. For simplicity, it is assumed that the latitude of the launching site and the latitude at which the satellite enters orbit are the same.

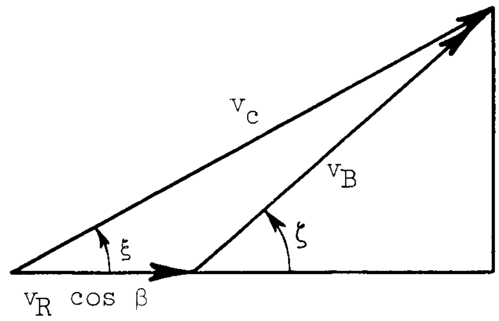
Consider the burnout velocity required to establish a satellite in a circular orbit with velocity v_c and angle of inclination $i > 0$. If the planet were not rotating, it would be possible to put the satellite into the given circular orbit from any launching site at latitude $\beta < i$ with the same burnout velocity. However, rotation of the planet introduces a linear velocity v_R to any satellite fired eastward from a launching site on the equator. For the Earth, v_R is approximately 1500 feet per second. At latitude β, the magnitude of this rotational velocity is $v_R \cos \beta$.

To launch a satellite from a site at latitude β into the circular orbit, the resultant velocity, a combination of burnout velocity and velocity due to the rotation of the planet, must be equal to the circular velocity v_c and make an angle ξ with the parallel of latitude in the eastward direction. This is shown in sketch (e):



(e)

The velocity vector diagram which indicates the required burnout velocity is shown in sketch (f). The required burnout velocity must be in a



(f)

direction making angle ζ with the eastward parallel of latitude and have a magnitude v_B. From sketch (f),

$$v_c \cos \xi = v_R \cos \beta + v_B \cos \zeta$$

$$v_c \sin \xi = v_B \sin \zeta$$

which gives

$$v_B^2 = v_c^2 - 2v_R v_c \cos \xi \cos \beta + v_R^2 \cos^2\beta \tag{B1}$$

and

$$\tan \zeta = \frac{v_c \sin \xi}{v_c \cos \xi - v_R \cos \beta} \tag{B2}$$

From spherical trigonometry,

$$\cos \xi = \frac{\cos i}{\cos \beta} \qquad \sin \xi = \frac{1}{\cos \beta} (\cos^2\beta - \cos^2 i)^{1/2} \tag{B3}$$

which gives

$$v_B^2 = v_c^2 - 2v_R v_c \cos i + v_R^2 \cos^2\beta \tag{B4}$$

$$\tan \zeta = \frac{v_c(\cos^2\beta - \cos^2 i)^{1/2}}{v_c \cos i - v_R \cos^2\beta} \tag{B5}$$

Examination of equation (B4) indicates that for a given $i > 0$ as β approaches i, v_B becomes smaller, and, therefore, the maximum burnout velocity is needed when the satellite is launched on the equator.

E-322

E-322

APPENDIX C

DERIVATION OF EQUATIONS FOR RENDEZVOUS BETWEEN CIRCULAR ORBITS

Equations (15) to (18) are derived as follows. The Δv for changing from orbit 1 to the transfer orbit is, from equations (8) to (10):

$$\begin{aligned}\Delta v_1 &= v_{T,1} - v_{c,1} \\ &= \sqrt{2\mu\left(\frac{1}{r_1} - \frac{1}{r_1 + r_2}\right)} - \sqrt{\frac{\mu}{r_1}}\end{aligned} \tag{C1}$$

Dividing through by $v_{c,1}$ and simplifying give

$$\begin{aligned}\Delta V_1 &= \sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \\ &= \sqrt{\frac{2\rho}{1 + \rho}} - 1\end{aligned} \tag{C2}$$

where $\rho = r_2/r_1$.

Likewise, to change from the transfer orbit to circular orbit 2:

$$\begin{aligned}\Delta v_2 &= v_{c,2} - v_{T,2} \\ &= \sqrt{\frac{\mu}{r_2}} - \sqrt{2\mu\left(\frac{1}{r_2} - \frac{1}{r_1 + r_2}\right)}\end{aligned} \tag{C3}$$

Dividing by $v_{c,1}$ and simplifying give

$$\begin{aligned}\Delta V_2 &= \sqrt{\frac{r_1}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right) \\ &= \sqrt{\frac{1}{\rho}}\left(1 - \sqrt{\frac{2}{1 + \rho}}\right)\end{aligned} \tag{C4}$$

During the time of transfer from the inner to the outer orbit, the outer satellite will travel through an angular distance of $180 - \Theta_1$ degrees. The angular distance is equal to the time in the transfer orbit times the rate of angular travel of the outer satellite:

$$180 - \Theta_{1,2} = \frac{P_T}{2}\frac{360}{P_2} \tag{C5}$$

Using equation (12) and simplifying,

$$\Theta_{1,2} = 180\left[1 - \left(\frac{r_1 + r_2}{2r_2}\right)^{3/2}\right]$$

or

$$\Theta_{1,2} = 180\left[1 - \left(\frac{1 + \rho}{2\rho}\right)^{3/2}\right] \tag{C6}$$

Likewise, during the time of transfer from the outer to the inner orbit, the inner satellite will travel through an angular distance of $180 + \Theta_{2,1}$ degrees and

$$180 + \Theta_{2,1} = \frac{P_T}{2}\frac{360}{P_1} \tag{C7}$$

$$\Theta_{2,1} = 180\left[\left(\frac{r_1 + r_2}{2r_1}\right)^{3/2} - 1\right]$$

$$\Theta_{2,1} = 180\left[\left(\frac{1 + \rho}{2}\right)^{3/2} - 1\right] \tag{C8}$$

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E-322

E-322

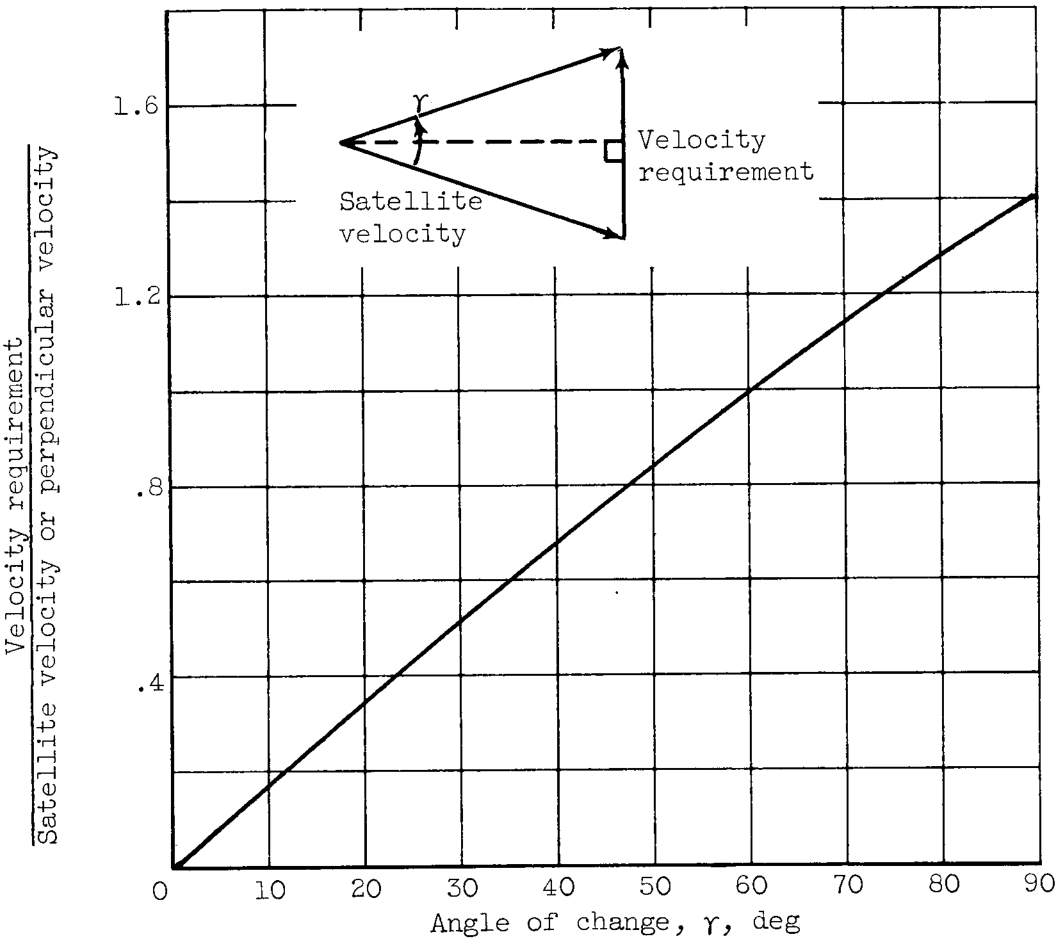

Figure 1. - Velocity requirement for changing plane or direction.

$$\frac{\text{Velocity requirement}}{\text{Satellite velocity}} = 2 \sin \frac{\gamma}{2}, \text{ direction change}$$

$$\frac{\text{Velocity requirement}}{\text{Satellite velocity perpendicular to radius}} = 2 \sin \frac{\gamma}{2}, \text{ plane change}$$

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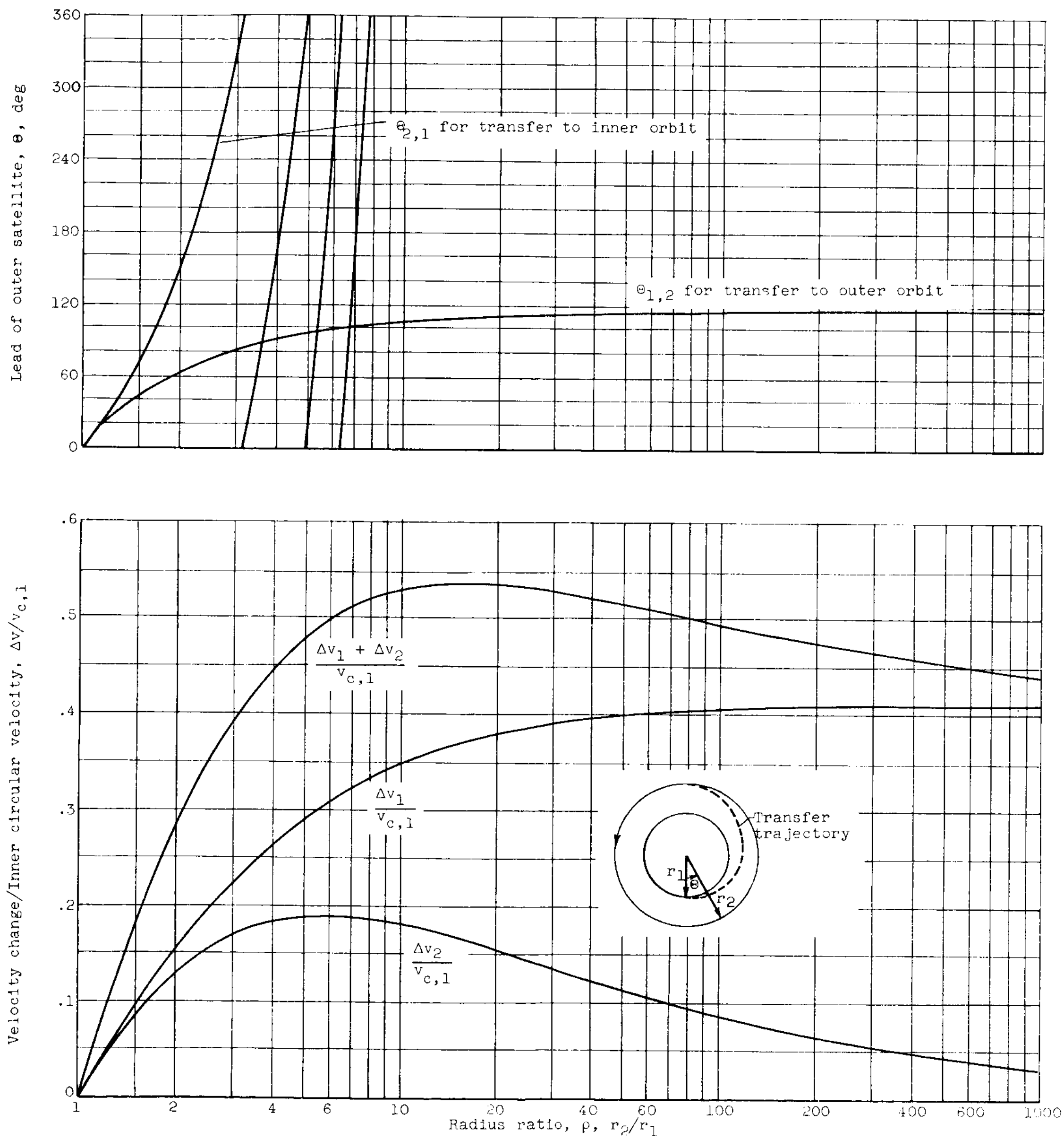


Figure 2. - Velocity changes and angular relation to rendezvous between circular orbits.

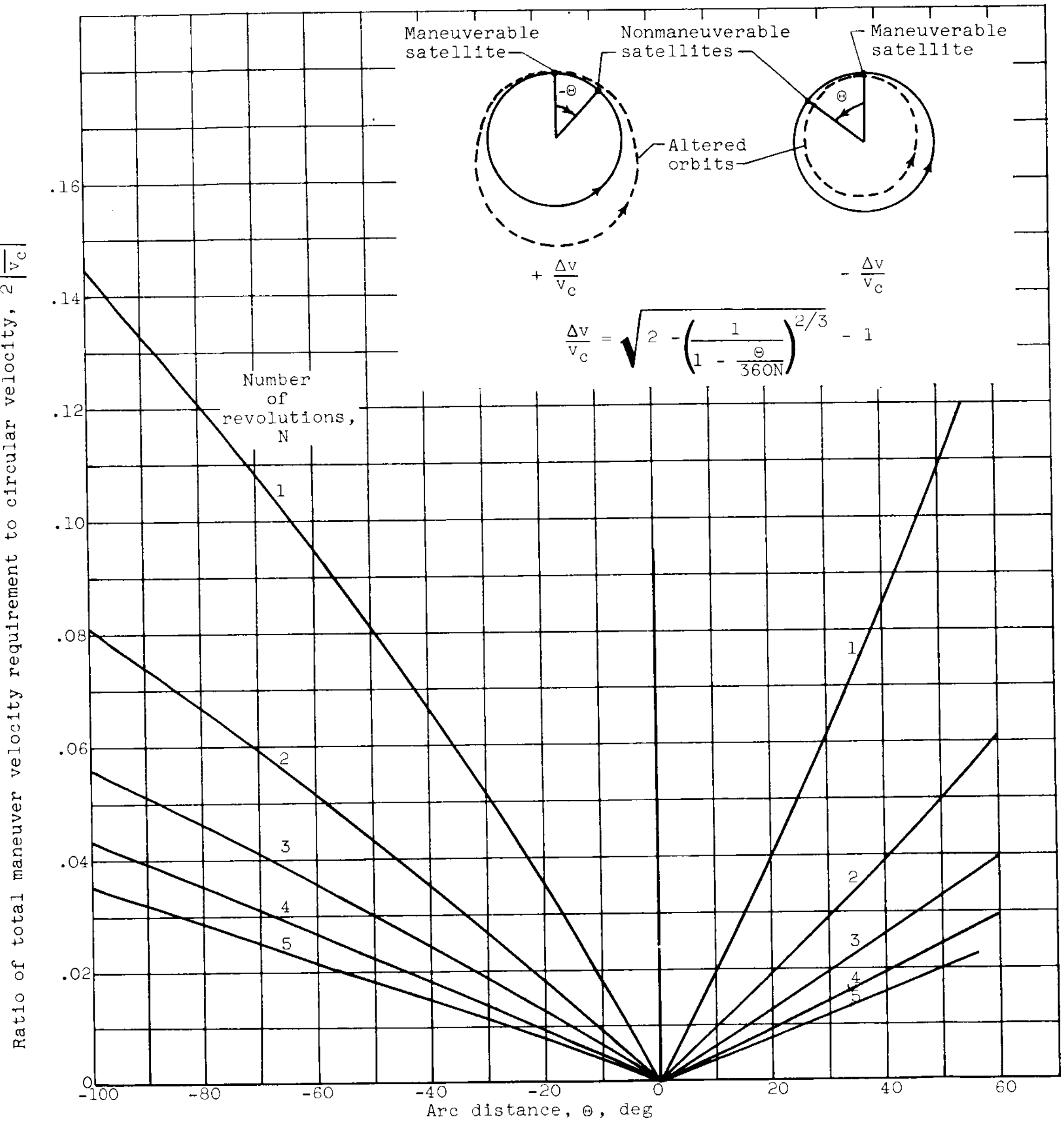


Figure 3. - Dimensionless plot for rendezvous of satellites in same circular orbit by altering the period.

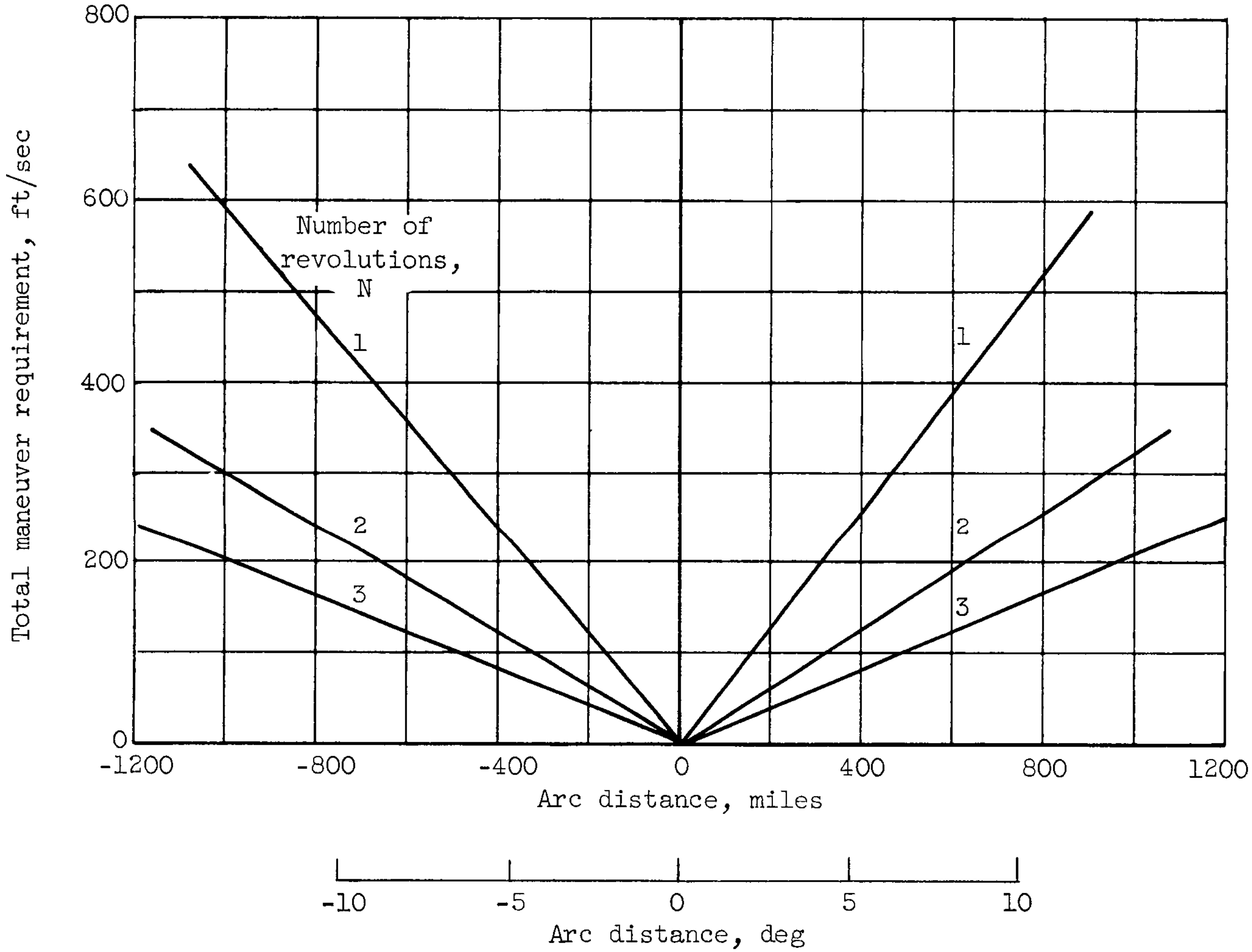


Figure 4. - Velocity requirement for rendezvous of satellites by altering the period (300-mile-altitude circular orbit about the Earth).

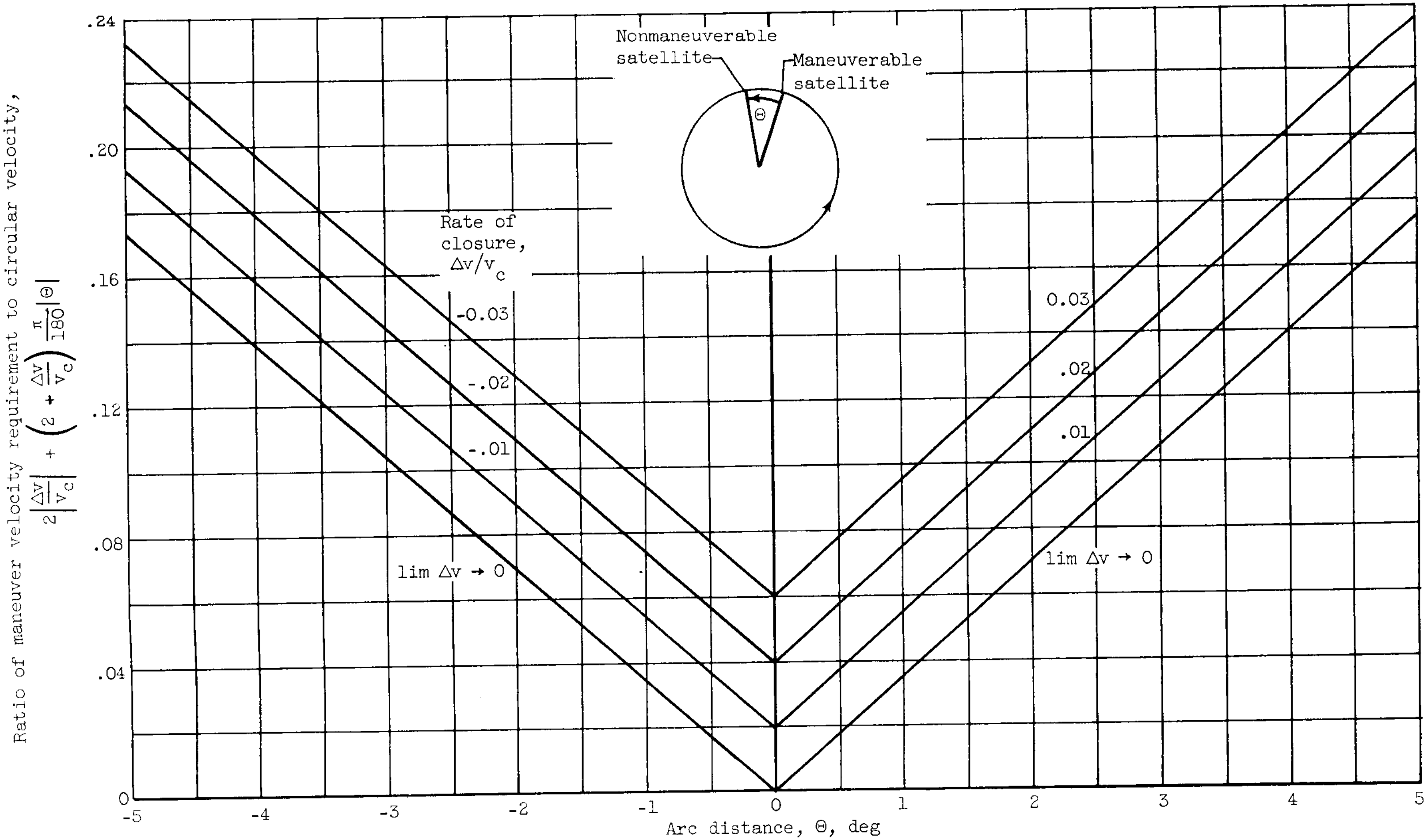


Figure 5. - Dimensionless plot for rendezvous of satellites in same circular orbit by balancing centrifugal force.

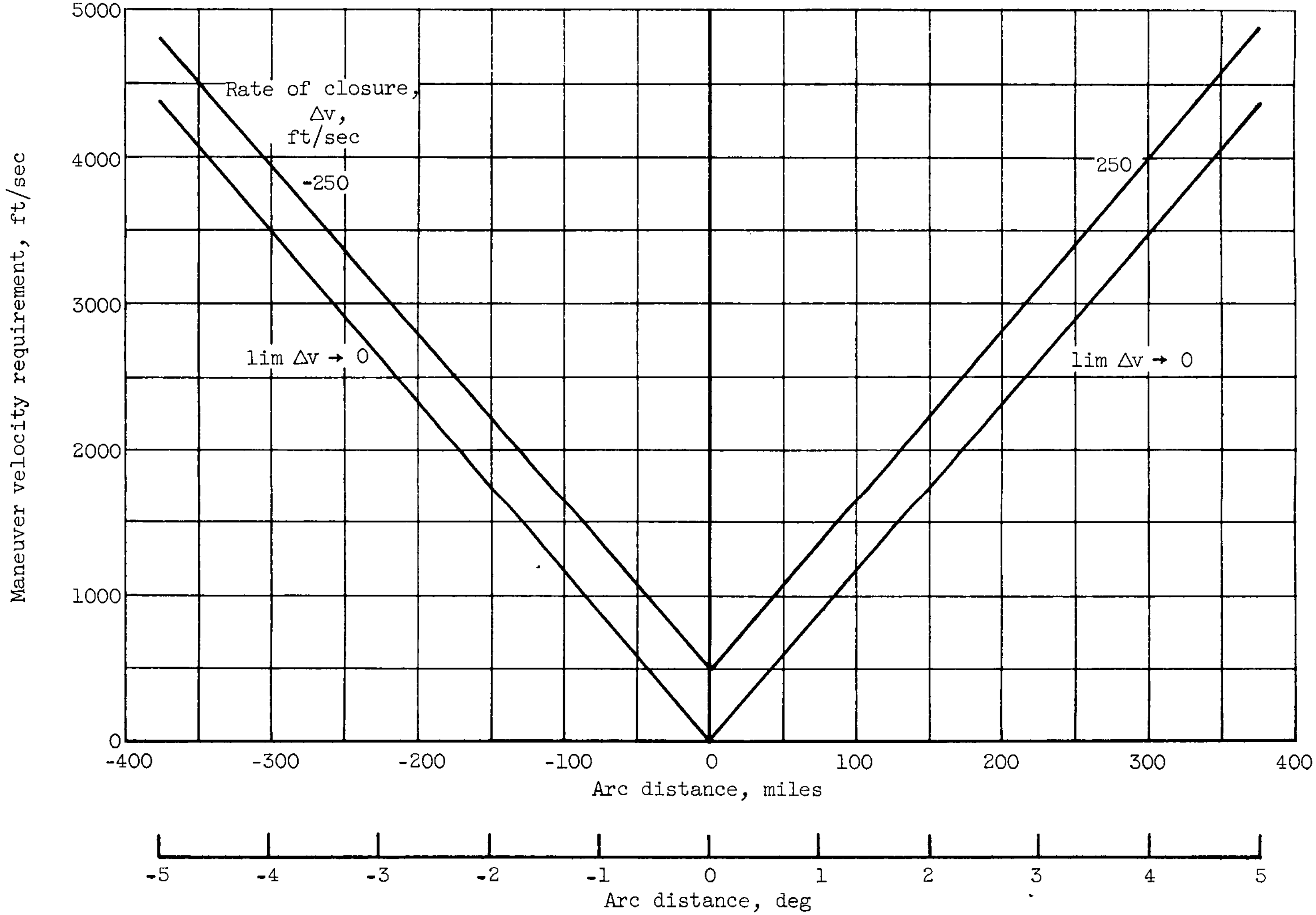
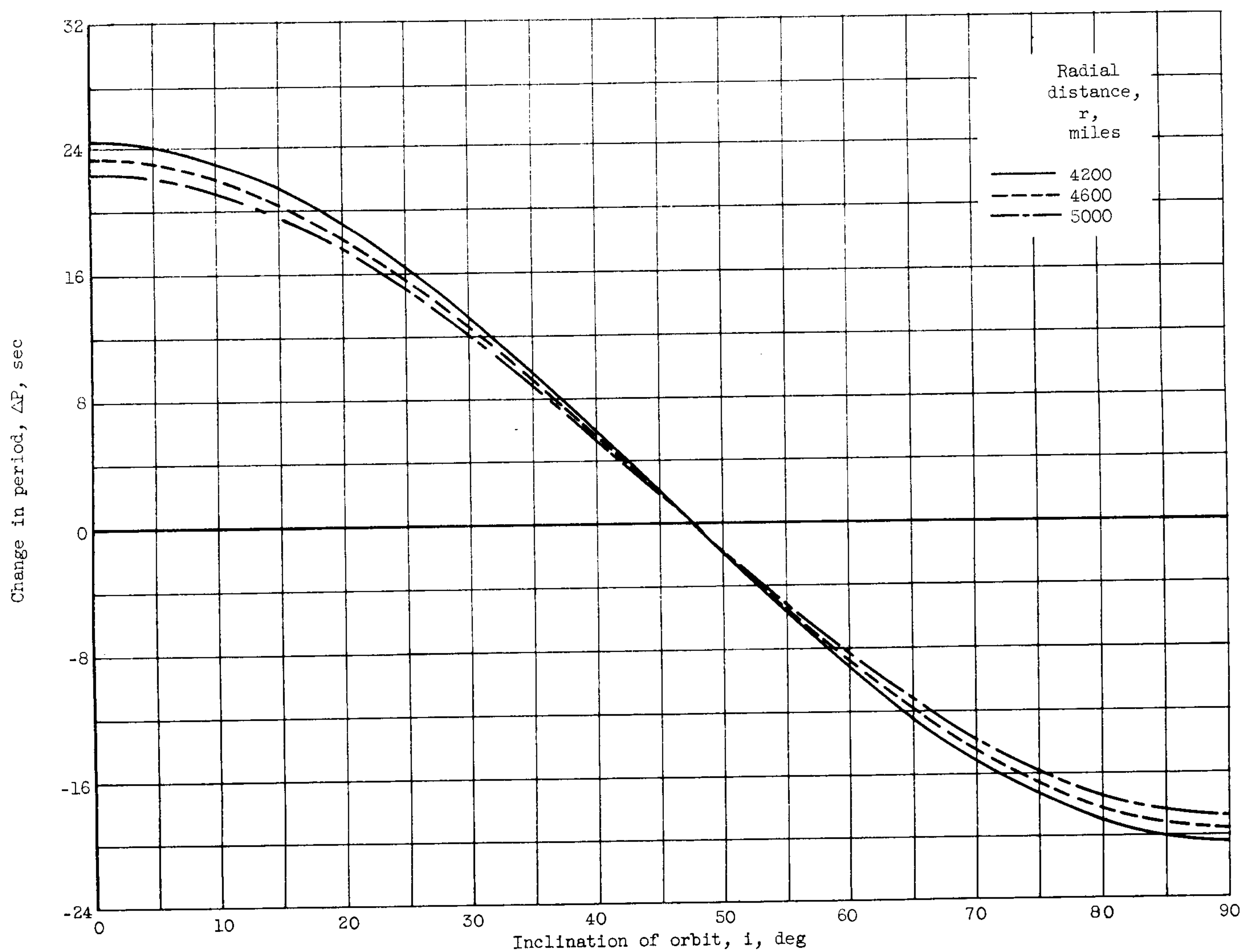
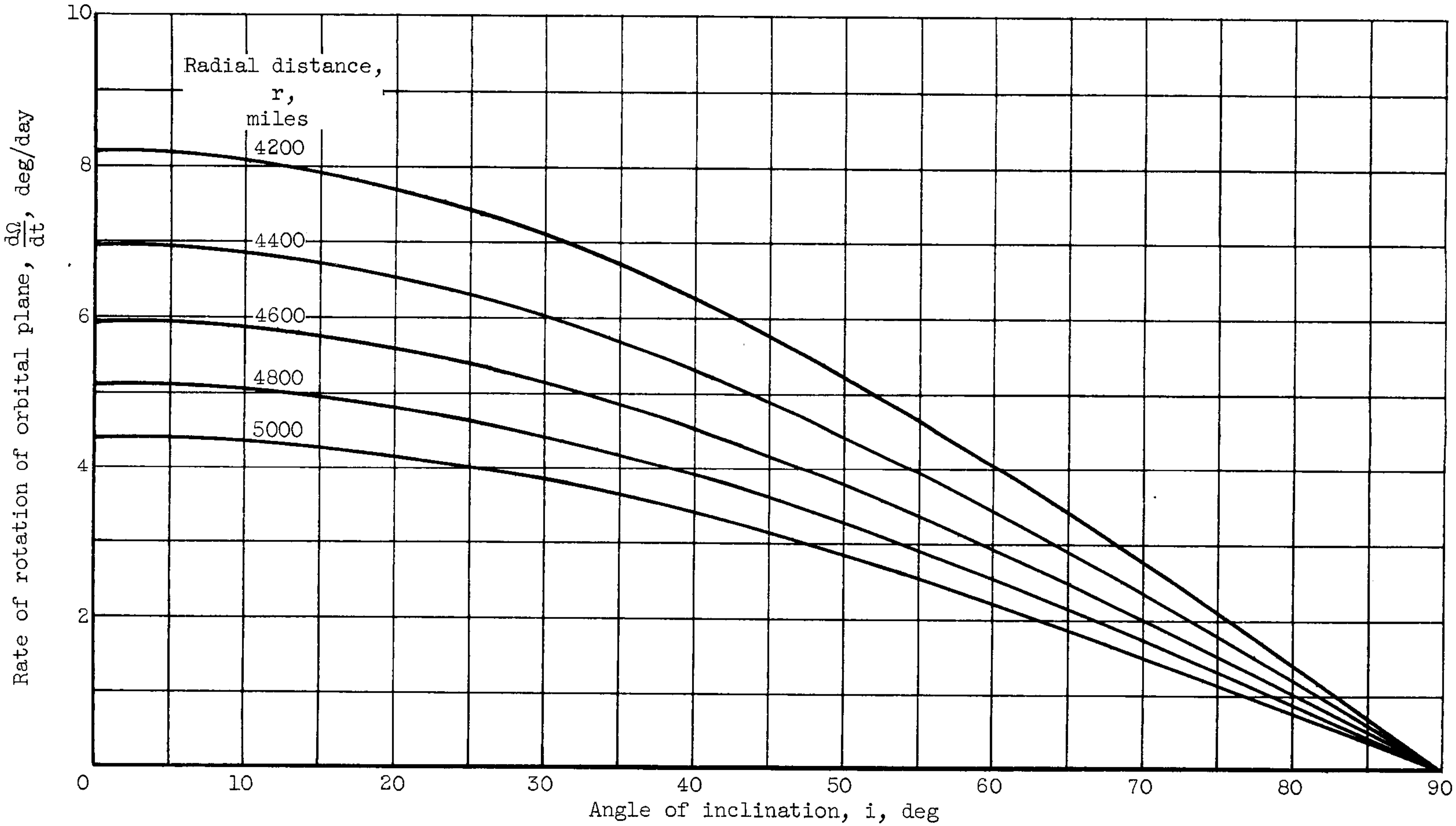


Figure 6. - Velocity requirement for rendezvous of satellites by balancing the centrifugal force (300-mile-altitude circular orbit about the Earth).



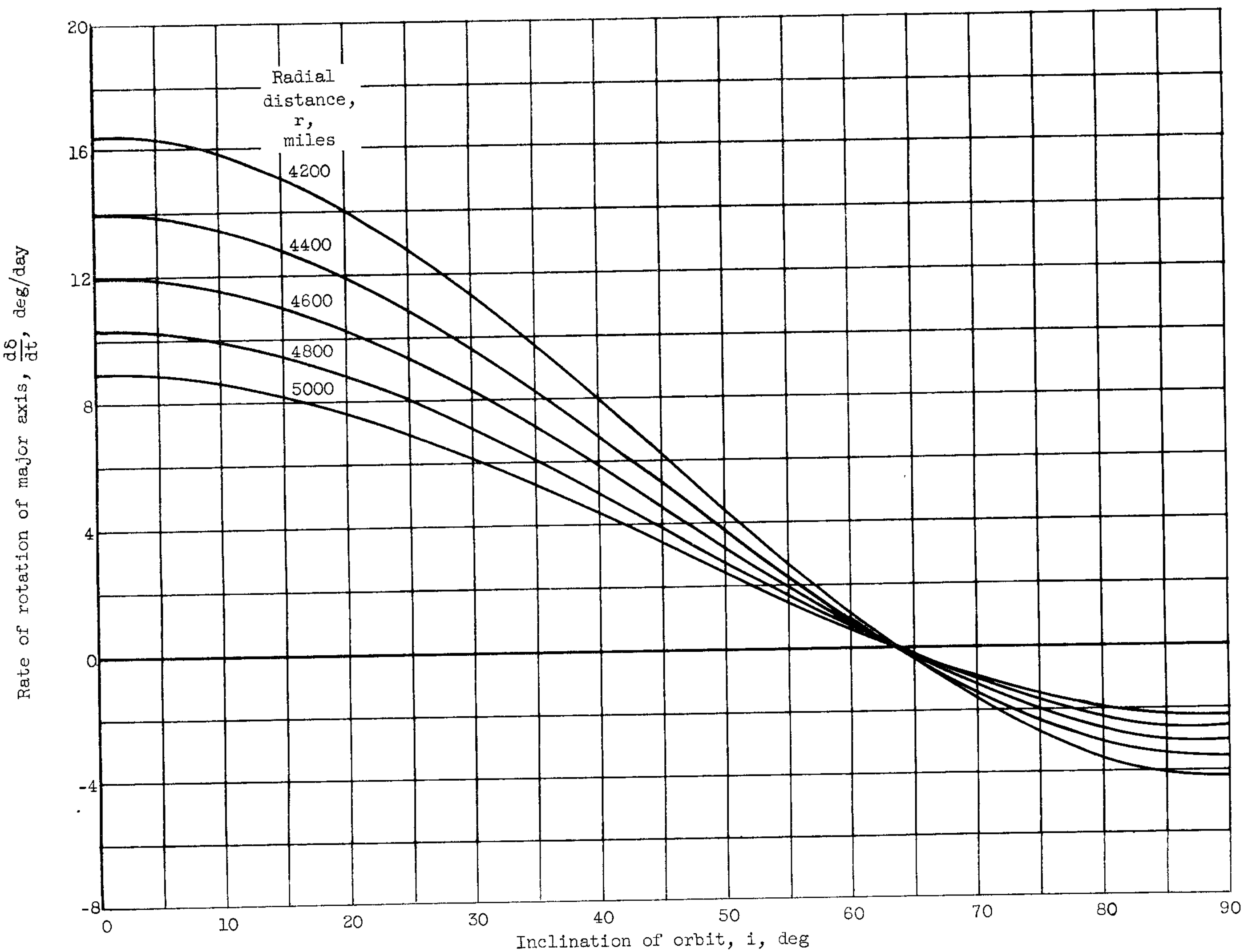
(a) Change in period.

Figure 7. - Variations in circular satellite orbit due to oblateness of Earth.



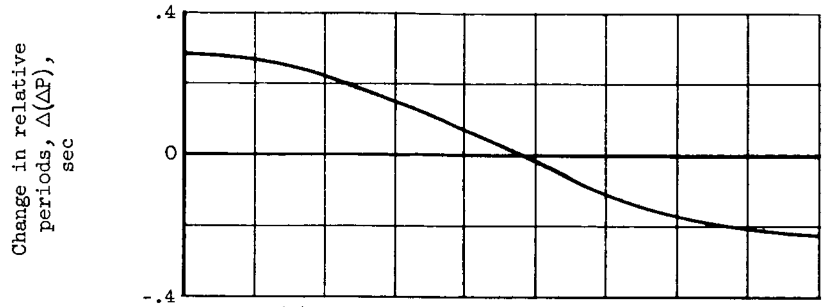
(b) Rate of rotation of orbital plane.

Figure 7. - Continued. Variations in circular satellite orbit due to oblateness of Earth.

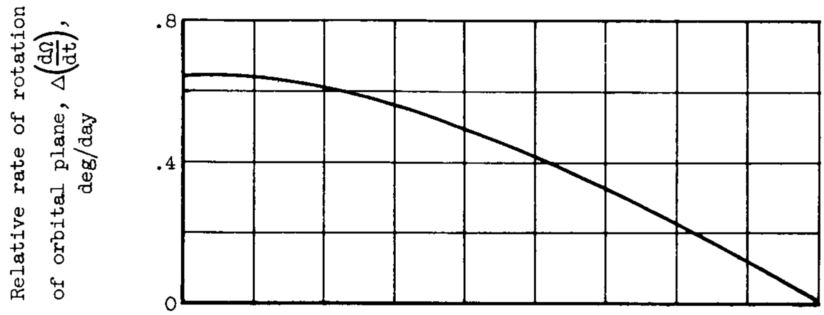


(c) Rate of rotation of major axis.

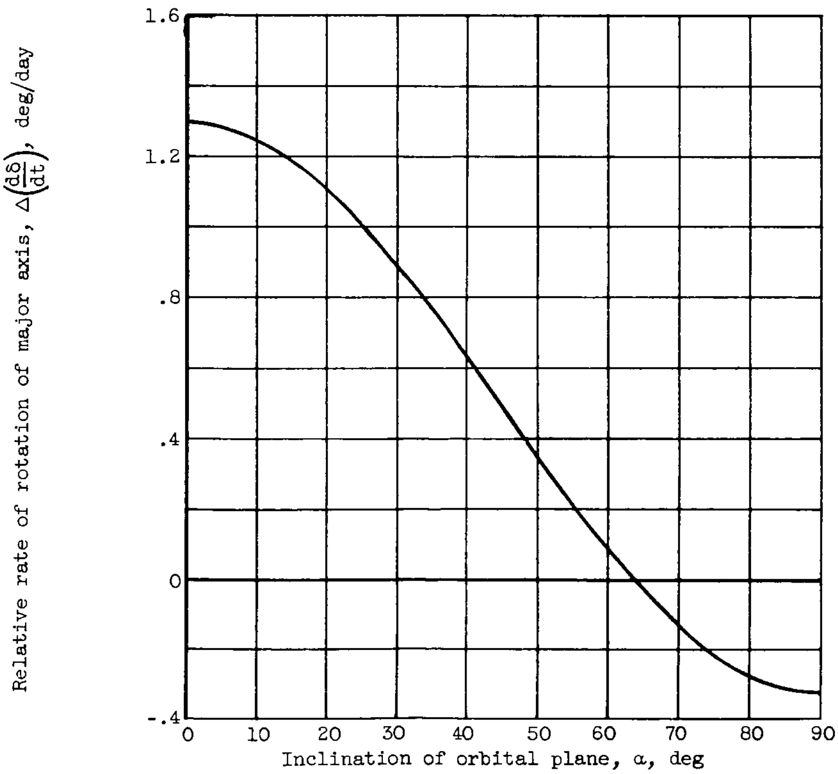
Figure 7. - Concluded. Variations in circular satellite orbit due to oblateness of Earth.



(a) Difference in variation of periods.



(b) Difference in rotation of orbital planes.



(c) Difference in rotation of major axes.

Figure 8. - Relative variations due to oblateness of Earth between a circular orbit at a radius of 4200 miles and a circular orbit at a radius of 4300 miles.

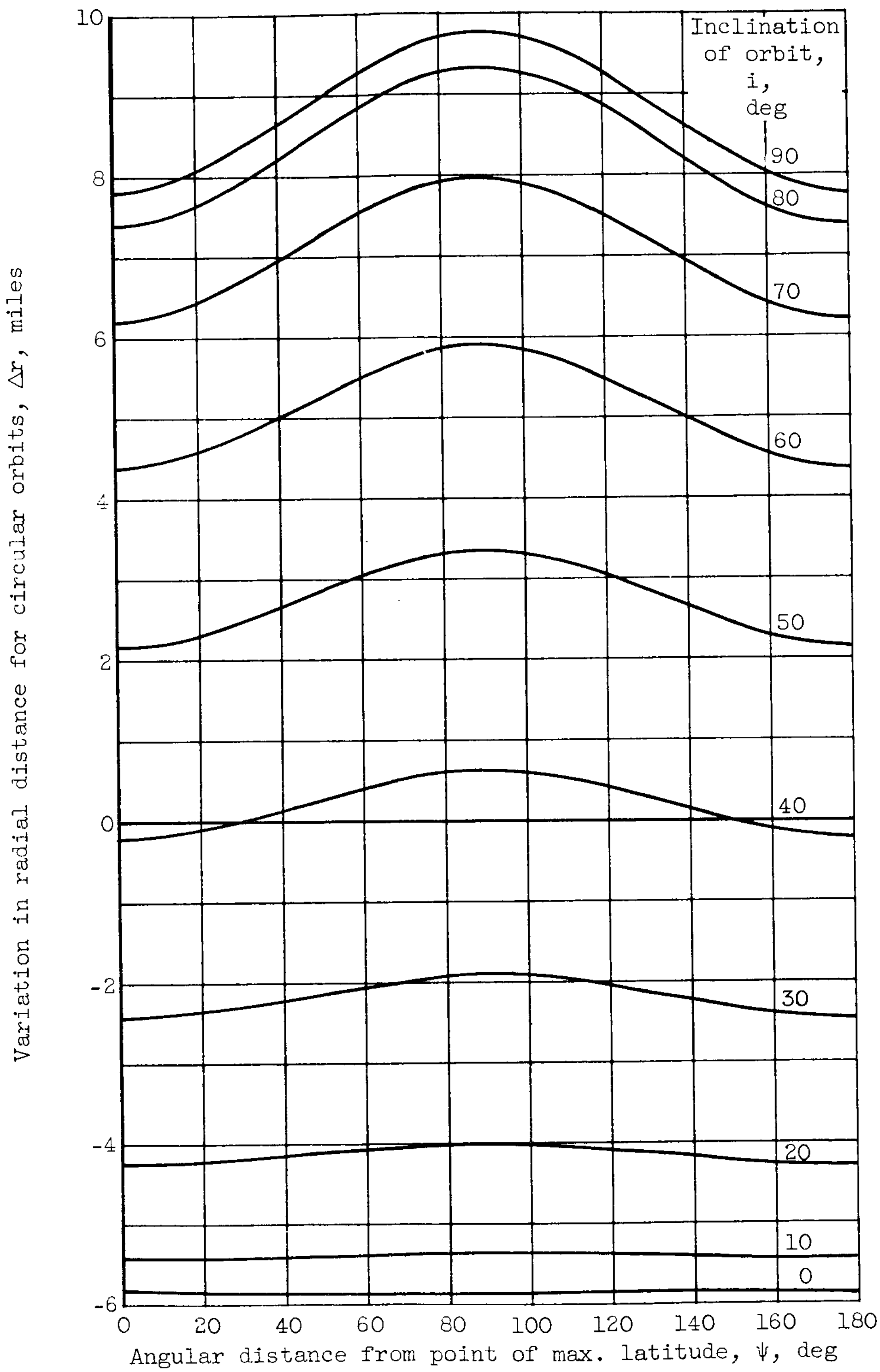


Figure 9. - Variation in radial distance due to oblateness of Earth for circular orbit at a nominal radial distance of 4400 miles.